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AIRBORNE RADAR SEARCH FOR DIESEL SUBMARINES

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ABSTRACT

Aircraft search to catch diesel submarines on the sea surface or with masts exposed above the sea surface has been an anti-submarine warfare tactic for more than half a century. However, rather than analysis, operational judgment has been used to guess at good search tactics such as how large an area one aircraft can cover effectively. In this research, a detection rate model is developed to analyze the effectiveness of airborne radar search for a diesel submarine assumed to be intermittently operating with periscopes or masts exposed above the sea surface. The analysis obtains cumulative probability of detection vs. time based on radar manufacturer's performance data, user inputs for aircraft search area size, search speed, and search altitude, and submarine periscope or mast exposure profiles. The model uses actual periscope radar cross section data, or roughly calculates radar cross section given assumptions about exposed periscope height above the sea surface and sea-state conditions. Submarine evasion due to radar counter-detection is also modeled.

PROBLEM DEFINITION

Diesel submarine detection by any sensor is a challenging problem. Active sonar is limited by short ranges, and passive sonar is limited by the quietness of submerged diesel submarines while they are operating on battery propulsion. A historically preferred tactic is airborne search, in which the searcher attempts to catch a submarine on the surface or with masts or periscopes exposed, briefly or intermittently, for battery charging, communications, or surface surveillance.

In the past, however, tactics have been based on operational judgment -- guesses about effective search area, search altitude, etc. The problem addressed in this paper is the development of an analytical model for Airborne Radar Search for Diesel Submarine (ARSDS). The purpose of the ARSDS model is to evaluate search tactics as a function of search area, searcher altitude, number of search aircraft, and other factors that can be varied at the discretion of the searcher; and then use the model as an aid to understanding effects of changes in submarine operating profile, submarine mast or periscope radar cross section, and other factors that are beyond the control of the searcher.

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CITATION OF RELATED WORK

The Tactics Development & Evaluation (TAC D&E) Program at the Navy Warfare Development Command (NWDC) motivated the problem of Airborne Radar Search for Diesel Submarines. The analytical model described in this paper was initially developed for use by Borges (2004) for a study of Radar Search and Detection using the CASA 212 S43 Aircraft by the Venezuelan Navy. A related study of airborne search for diesel submarines was conducted by Monfore (2004, unpublished) using an agent-based simulation model, unrelated to the analytical model. NWDC is now developing a Tactics Memorandum (TACMEMO) based on these studies. The basic idea of detection rate models, including the famous random search model, was used by Koopman (1946) during World War II, and is found in many textbooks, such as Wagner, et. al. (1999) and Washburn (2002).

DESCRIPTION OF APPROACH

The ARSDS model is an equation-based, analytical detection rate model – not a simulation. The principal measure of effectiveness for ARSDS is the probability of radar detection of a submarine that is only detectable during intermittent periods of periscope exposure. Probability of detection is obtained from a detection rate over time. The detection rate is derived from the rate at which submarine periscope exposures occur, and the probability that the search aircraft radar happens to be covering that patch of ocean during those intermittent periscope exposures, and the probability that the submarine is unable to avoid detection by the aircraft due to earlier counter-detection of the radar. These rates and probabilities are obtained from parameters of the model which include the expected frequency and duration of submarine operations with periscopes or masts exposed, the submarine periscope and/or mast radar cross section (RCS), the search aircraft speed, altitude, and search area, and radar detection ranges considering periscope RCS, search altitude, and sea-state conditions.

STATEMENT OF ASSUMPTIONS

The model assumes there is a submarine operating somewhere within an area of interest in which airborne radar search is conducted. The submarine position at any instant is equally likely to be anywhere in the area, i.e., target location is assumed to be uniformly distributed. It is assumed that the area of interest is large compared to the much smaller patch of ocean that the search radar covers in any short increment of time, and that the probability of making a detection in any short time interval is probabilistically independent of detections in non-overlapping time intervals. Submarine speed is very much slower than aircraft speed, so that in the time it takes for an airborne radar patch to sweep by an exposed submarine, the submarine position can be treated as stationary.

A critical assumption of this ARSDS model is that there are intermittent periods of periscope exposure, and that either intelligence information or expert judgment can be used to at least estimate the expected frequency and duration of submarine operations with periscopes or masts exposed.

The model does not require any restrictive or simplifying assumptions concerning submarine periscope radar cross section, or search radar detection ranges. Values based on actual data can be used, and are preferred. In the absence of some actual data, however, some simple assumptions are used to generate placeholders for periscope radar cross section and search radar lateral range functions, as described in the following explanation of methodology.

Furthermore, the model does not restrict consideration of a single periscope or mast configuration. The detection rate methodology allows for computation of detection rates versus any number of periscopes and/or masts, with differing radar cross sections and differing exposure rates. In addition, the model does not require an assumption of a constant detection rate – the model is easily adapted to detection rates that vary by time of day.

EXPLANATION OF METHODOLOGY

Search Theory Review – Detection Rate Models

Detection rate models are used for modeling probability of detection for continuous-looking search. The basic idea is that the detection process is a Poisson Process that assumes that detections in non-overlapping time intervals are independent, and that the expected number of detections in very small time intervals is approximately equal to the product of the detection rate in that very small interval multiplied by the length of the small interval. If the detection rate is a constant, often denoted λ , then the time between detections is an exponential random variable with mean $1/\lambda$, and the number of detections in time t is a Poisson random variable with mean λt . The cumulative detection probability for a search from time 0 up to time t , $CDP(t)$ defined as the probability of at least one detection up to time t is

$$CDP(t) = P\{1 \text{ or more det in time } t\} = 1 - e^{-\lambda t} \quad (1)$$

If the detection rate varies with time, i.e., there is a variable detection rate $\gamma(t)$, then the detection process is called a non-homogeneous Poisson Process, and cumulative detection probability up to time t is

$$CDP(t) = P\{1 \text{ or more det in time } t\} = 1 - e^{-\int_0^t \gamma(s) ds} \quad (2)$$

Examples of detection rate models are the Inverse-cube Law of Sighting for visual search, the Poisson scan model for sonar search, the blip-scan model for radar search, and the very well-known random search model. In random search, the detection rate is usually argued to be a constant $v w / A$, where the target is stationary and uniformly distributed in area A , v is the search speed, w is sensor sweep width. For random search, cumulative detection probability up to time t is

$$CDP(t) = P\{1 \text{ or more det in time } t\} = 1 - e^{-\frac{vw}{A}t} \quad (3)$$

It is important to note that whether the detection rate is constant or varies with time, the term in the exponent of equations (1) or (2) is simply the area under the detection rate curve in time t . Hence, the key to detection rate models is coming up with a detection rate.

The detection rate derived in the following sections happens to be a constant detection rate vs. a single periscope or mast target. This permits a clearer explanation of the model, but is not required for computation. Computations are performed numerically in a spreadsheet or with other computational software, and the model can be extended to multiple periscopes and/or masts with varying exposure profiles by time of day, such that the detection rate ends up being quite non-homogeneous.

ARSDS Detection Rate

The idea for the ARSDS detection rate is that the rate at which detections can be made is governed by the rate at which occasional periscope exposures occur. Then, when an exposure occurs, it can result in detection if the searching aircraft radar happens to be covering the patch of ocean where the submarine periscope happens to be, and the submarine does not get a chance to evade due to radar counter-detection. This is summarized as

$$\text{ARSDS Detection Rate} = \left[\begin{array}{c} \text{Rate of} \\ \text{occurrence} \\ \text{of submarine} \\ \text{periscope} \\ \text{exposures} \end{array} \right] * P \left[\begin{array}{c} \text{Aircraft radar} \\ \text{detection patch} \\ \text{is covering spot} \\ \text{when periscope} \\ \text{exposure occurs} \end{array} \right] * P \left[\begin{array}{c} \text{Submarine does} \\ \text{not avoid} \\ \text{detection} \\ \text{due to radar} \\ \text{counter-detection} \end{array} \right] \quad (4)$$

The terms in the detection rate are developed from a combination of data, basic radar principles, and search theory.

Submarine Periscope or Mast RCS and Sea-state Degradation of Radar Return

If actual target radar cross section (RCS) under various sea-state conditions is known, those actual values can be used in calculating the factors in the detection rate. This would be preferred. In the absence of actual data, however, fundamental radar theory is used to compute a normal target radar cross section based on assuming the periscope mast is a simple shape exposed a specified height above the sea surface. For example, the physics-based formula for a normal reflection from a cylindrical target RCS is $2\pi r l^2 / \lambda$, where r is the radius of the cylinder, l is the height of the cylinder, and λ is the wavelength of the search radar (Knott, 1993). The formula provides peak radar cross section for radar on an axis perpendicular to the cylinder. It can also apply if it is assumed that the cylinder is perpendicular to a perfectly smooth corner reflector as might be the case with a very smooth sea surface. However, degradation due to less than perfect reflection caused by sea surface waves is to be expected, and roughly approximated by an assumed percent reflection table as a function of sea-state, such as Table 1.

As described, this approach to roughly approximating target RCS is a placeholder for actual target RCS data. This approach does model the physical size of the target, the radar wavelength, and sea-state, but does not include such factors as stealth design, which might tend to reduce RCS, nor periscope wake effects, which could increase RCS. A more complex placeholder approximation could be developed.

Sea-state	Description	Assumed % reflection
0	flat surface	100
1	smooth	90
2	slight	75
3	moderate	50
4	rough	15
5	very rough	2.5
6	High	1
7	very high	.15
8	mountainous	.07
9	very mountainous	.01

Table 1. RCS reflection due to sea-state (example)

Search Radar Maximum Range and Footprint

The ARSDS model uses actual data on search radar capability provided by the manufacturer, which should generally be available to the searcher. For airborne search for a small RCS target, maximum radar range, as determined by the radar range equation, is generally the range at which the radar return from the target falls below the level that the radar receiver can distinguish from noise (Skolnick, 2001, or Stimson, 1998). Typical manufacturer's data will give a maximum range based on target RCS, aircraft altitude, and radar settings. For example, Landa (2004) obtained manufacturer data for the RDR-1500B search radar on the Venezuelan Navy CASA 212 S43 maritime patrol aircraft, an example of which is shown in Figure 1.

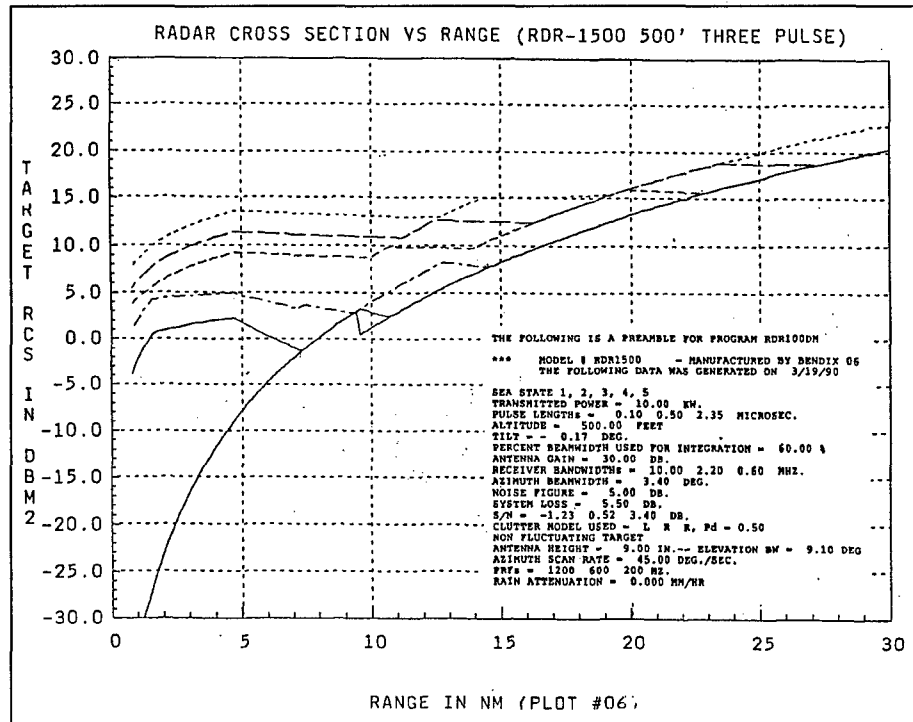


Figure 1. RDR 1500 Radar Range vs. Target RCS (example)

Radar Sweep Width and Lateral Range Function

Sweep width for the radar when flown at a particular altitude searching for a target of a particular RCS is needed for computing the detection rate. Two options exist for determining sweep width.

Option One would assume the radar footprint acts like a cookie-cutter and thus the overall width of the footprint would be the sweep width. The following discussion describes the reasoning behind this method and concludes that it is not used due to some shortcomings. Since the radar footprint was determined based upon the radar ability to see targets within that footprint (and conversely its inability to see targets outside the footprint), the radar footprint could possibly be interpreted as a cookie-cutter detection pattern (i.e., detecting every target that falls within the footprint with probability 1). Such a cookie-cutter sweep width might be overly optimistic in practice because of the irregular shape of the radar footprint. In fact, as the radar footprint sweeps over area, points close to the extreme left and right corners of the pattern are within the footprint for much less time than points that are passed closer to the middle of the pattern. Accordingly, it is deemed needlessly unrealistic to assume the full width of the radar footprint is a cookie-cutter sweep width, and therefore this method is not used.

Option Two is to calculate sweep width as the integral of the lateral range function over all possible closest points of approach between the aircraft and the submarine, i.e., find the area under the radar lateral range curve (Koopman, 1946). This is the preferred method that is used. If actual lateral range curves for the radar were available from the manufacturer, or from operational testing, they could be used directly. As a placeholder for actual data, an approximate lateral range function can be derived based on the geometry of the radar footprint.

Lateral Range Function

Lateral range is the closest point of approach (CPA) between the searcher and the target assuming an infinitely long straight-line relative motion path. The lateral range function, $F_L(x)$, is a cumulative detection probability as a function of the lateral range x (Koopman, 1946). These definitions implicitly assume that a target exists that can be detected. In this context, the target would be an exposed submarine periscope. Accordingly, the cumulative probability of detection used in the lateral range function might more correctly be called a conditional cumulative probability of detection given that the submarine periscope is exposed. This is very significantly different from the cumulative detection probability that is ultimately computed based on intermittent submarine periscope exposure and counter-detection evasion.

Lateral Range Function Derivation

Given an exposed target, detection depends on the maximum detection range (R_{\max}), the amount of time an exposed target would be inside the radar footprint, and whatever the detection rate is for an exposed target. When CPA range $x \leq R_{\max}$, the target could be detected and when CPA range $x > R_{\max}$ the target is not detectable. The target enters in the area of possible detection at point (x, y_0) . The location of the target at time t is $(x, y(t)) = (x, y_0 - vt)$, where v is the relative speed. Submarine speed is very slow compared to aircraft search speed and thus relative speed is approximately just the aircraft speed. The target reaches CPA at time $t = y_0 / v$ and moves out of the area of detection.

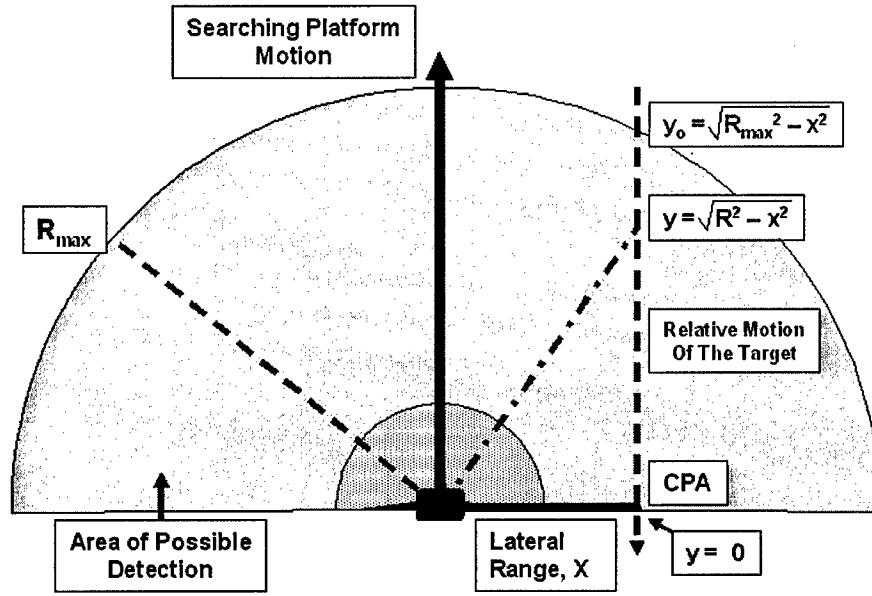


Figure 2. Lateral Range Function Derivation

Wagner (1999) derives a lateral range function for a situation comparable to the situation here. If it is assumed that the radar footprint passes over an area containing an exposed submarine periscope, and that during this encounter a constant detection rate applies, then the lateral range function takes the following form, where K is a constant.

$$F_L(x) = 1 - e^{\left(-K \sqrt{R_{\max}^2 - x^2} / v\right)} \quad (5)$$

for $x \leq R_{\max}$.

Wagner's result, equation (5), can be applied to approximate a placeholder lateral range function for ARSDS. The maximum value of this lateral range function, when CPA range $x = 0$, is

$$P_{\max} = 1 - e^{\left(-K R_{\max} / v\right)} \quad (6)$$

From equation (6) an expression is obtained for the constant K .

$$K = -\left(\frac{v}{R_{\max}}\right) \ln(1 - P_{\max}) \quad (7)$$

Let $q_0 = 1 - P_{\max}$, the probability that the target is not detected by radar when CPA range $x = 0$, i.e., when the aircraft flies on a path that passes directly over an exposed periscope. Then the lateral range function, equation (5), simplifies to

$$F_L(x) = 1 - q_0 \sqrt{1 - \left(\frac{x}{R_{\max}}\right)^2}$$

for $x \leq R_{\max}$. The parameter q_0 can now be used as a scaling parameter for this approximate lateral range function, where q_0 is interpreted as a realistic probability that due to some factors not explicitly modeled, the search aircraft may fly directly over an exposed periscope and still

not see it. Ideally, an estimate for q_0 could be obtained from operational data. An example of this approximate lateral range function, for $R_{\max} = 14.0$ nm and $q_0 = .05$, is shown by the solid curved line in figure 3.

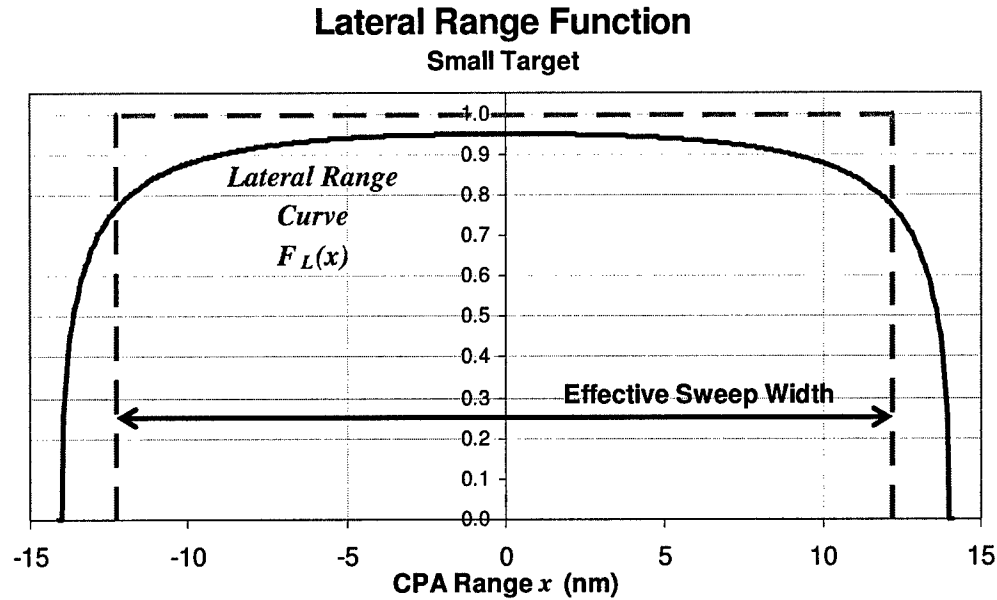


Figure 3. Lateral Range Function Example. $R_{\max} = 14.0$ nm, $q_0 = .05$

Effective Sweep Width

Sweep width, w is defined as the area under the lateral range curve,

$$w = \int_{-R_{\max}}^{+R_{\max}} F_L(x) dx \quad (8)$$

For the example, $R_{\max} = 14.0$ nm and $q_0 = .05$, effective sweep width, $w = 24.5$ nm, calculated by numerical integration. It is common to also think about a cookie-cutter sensor that has the same sweep width as the radar, which in some circumstances may provide equivalent performance (Washburn, 2002). The lateral range function of the “equivalent” cookie-cutter sensor is rectangular, with height 1.0 and width w . This interpretation corresponds to the common understanding about sweep width representing a definite swath of detection swept out by the sensor. The dashed rectangular lines in figure 3 show the lateral range function of the “equivalent” cookie-cutter sensor.

Swept Radar Patches and Radar Glimpses

For convenience, it is considered that the radar lays down a pattern of non-overlapping patches, each considered a radar glimpse. The radar patch is of width w . By definition sweep rate = sweep width * aircraft search speed = wv (Koopman, 1946). The length of each swept radar patch is arbitrary. A convenient patch length = $R_{\max} - R_{\min}$, where R_{\max} , as already described, is based on the radar range equation, and R_{\min} for an airborne search radar is typically due to radar altitude and the depression angle of the radar down to the surface (Stimpson, 1998). For example, for the RDR-1500B search radar on the Venezuelan Navy CASA 212 aircraft, at 500 foot altitude, $R_{\min} = .87$ nm on the nose of the aircraft.

A Radar Glimpse Interval is now defined as the time it takes the aircraft to fly over one swept radar patch. The radar glimpse interval is thus calculated as

$$\text{Radar Glimpse Interval (hrs)} = \frac{\text{Radar Patch Length (nm)}}{\text{Aircraft Search Speed (kts)}} \quad (9)$$

The area of a radar patch is the increment of search area swept by the search aircraft in one glimpse interval.

$$\text{Radar Patch Area} = \left(\begin{array}{c} \text{Radar} \\ \text{Glimpse} \\ \text{Interval} \end{array} \right) * \left(\begin{array}{c} \text{Effective} \\ \text{Sweep} \\ \text{Rate} \end{array} \right) \quad (10)$$

Conditional glimpse p_d given periscope exposure, $p_{d|exposure}$, is defined as the likelihood that the relatively small aircraft radar patch happens to be covering the point in the much larger search area, A , when a detection opportunity (i.e., periscope exposure) occurs, assuming that the uncertain submarine position, when exposed, is equally likely to be anywhere in the search area, A .

$$p_{d|exposure} = \frac{\text{Radar Patch Area}}{\text{Search Area}} \quad (11)$$

Periscope Exposure Rate

As previously discussed, it is assumed that either intelligence information or expert judgment can be used to at least estimate the expected frequency and duration of submarine operations with periscopes or masts exposed, from which a periscope exposure rate can be calculated. An Operational Period, an input to the model, is defined as any convenient fixed time period used to summarize the submarine operating profile, such as a 24-hour day. The operational period includes time spent completely submerged and time spent with periscopes or masts exposed for any purpose. Periscope Exposure Hours, also an input to the model, is the expected amount of time during each Operational Period that the submarine has periscopes or masts exposed for any purpose such as recharging batteries, communicating, or conducting surveillance. A Glimpse Count can then be calculated to get the number of glimpse intervals that comprise Periscope Exposure Hours during each Operational Period.

$$\text{Glimpse Count} = \frac{\text{Periscope Exposure Hours (hrs)}}{\text{Radar Glimpse Interval (hrs)}} \quad (12)$$

With this, a periscope exposure rate can be calculated.

$$\text{Periscope Exposure Rate (hrs}^{-1}\text{)} = \frac{\text{Glimpse Count}}{\text{Operational Period (hrs)}} \quad (13)$$

As was previously noted, and described here, this version of the model computes a constant periscope exposure rate or detection opportunity rate. The model could be easily adapted to allow for an opportunity rate that varies by time of day, for example.

ARSDS Detection Rate

So far, neglecting radar counter-detection

$$\text{ARSDS Detection Rate} = \left[\frac{\text{periscope exposure rate}}{\text{rate}} \right] * p_{d|\text{exposure}} \quad (14)$$

Using the terms defined in the preceding paragraphs

$$\text{ARSDS Detection Rate} = \frac{\left(\frac{\text{Periscope Exposure Hours}}{\text{Radar Glimpse Interval}} \right)}{\text{Operational Period}} * \frac{\left(\frac{\text{Radar Glimpse Interval}}{\text{Search Area}} \right) * \left(\frac{\text{Effective Sweep Rate}}{\text{Rate}} \right)}{\text{Rate}} \quad (15)$$

This simplifies to

$$\text{ARSDS Detection Rate} = \left(\frac{\text{Periscope Exposure Hours}}{\text{Operational Period}} \right) * \frac{v w}{A} \quad (16)$$

The term in parentheses can be interpreted as either a percentage of time that the submarine periscopes are exposed, or equivalently, the probability that a submarine might be exposed at any instant in time, p_{exposure} . Thus the ARSDS detection rate, neglecting radar counter-detection is

$$\text{ARSDS Detection Rate} = p_{\text{exposure}} * \frac{v w}{A} \quad (17)$$

In this form, it may be said that **the ARSDS detection rate is a fraction of the random search model detection rate**. Although this simple relationship was not anticipated in the preceding derivation, it is not altogether surprising that this should turn out to be the case. Washburn (2002) describes an experiment in which both searcher and target movements are unknown to each other within a fixed area. The data from repetitions of the experiment show that time to detection has the same properties as for the random search model, i.e., time to detection in both cases is exponentially distributed. The result, reflected in equation (17), provides insight concerning ARSDS. The random search detection rate, $v w / A$, can be interpreted as the rate at which the aircraft radar could detect a submarine in randomly distributed patches if it were exposed, and p_{exposure} is the fraction of those encounters for which the submarine is exposed above the surface, and thus detected.

Radar Counter-detection by the Submarine

The ARSDS model considers the possibility that the search radar emission can be counter-detected by the target submarine at longer ranges than the radar can see the much smaller reflections from the submarine masts or periscopes. Thus, the submarine may have an opportunity to submerge and avoid radar detection.

If the submarine periscope exposure occurs in the area inside the radar R_{\max} , then the submarine will be detected before it can avoid. If the submarine periscope exposure occurs in the area outside the radar R_{\max} , but within the radar horizon, R_h , then the submarine can detect the radar emission and submerge to avoid detection. This geometry is illustrated in figure 4.

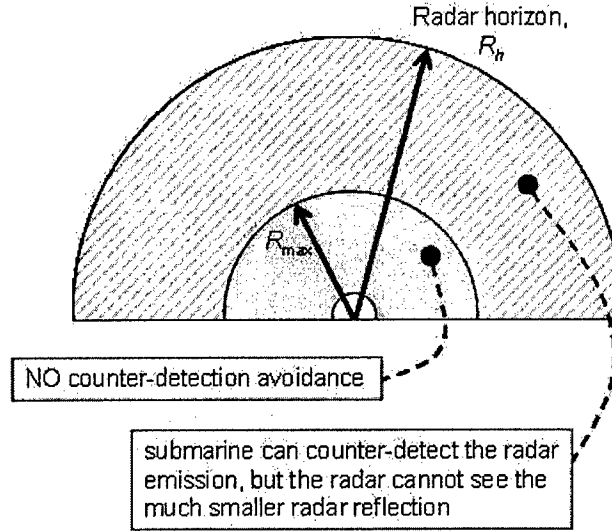


Figure 4. Radar Counter-detection by the Submarine

The conditional probability that a submarine within the search aircraft radar horizon does not get the chance to avoid detection due to radar counter detection is modeled as the ratio of the detection area to the radar horizon area, or

$$P \left[\begin{array}{l} \text{Submarine does not} \\ \text{avoid detection due to} \\ \text{radar counter-detection} \end{array} \right] = \frac{R_{\max}^2}{R_h^2} \quad (18)$$

where $R_{\max} \leq R_h$, and 1 otherwise.

ARSDS Detection Rate Model

With radar counter-detection considered, the ARSDS detection rate is

$$\text{ARSDS Detection Rate} = \frac{v w}{A} * p_{\text{exposure}} * \frac{R_{\max}^2}{R_h^2} \quad (19)$$

Then the ARSDS Cumulative Detection Probability as a function of time t , is

$$CDP(t) = P\{1 \text{ or more det in time } t\} = 1 - e^{-\frac{v w}{A} P_{\text{exposure}} \frac{R_{\text{max}}^2}{R_h^2} t} \quad (20)$$

Example

For the RDR-1500B search radar on the Venezuelan Navy CASA 212 S43 maritime patrol aircraft, for three different exposed periscope heights of .5, .6, .7 meters, respectively, exposed a total of 6 hours per day, in sea-state 1, in a search area of 60 x 60 nm, aircraft altitude of 500 feet, and search speed of 180 knots, figure 5 shows the resulting cumulative detection probability as a function of search time.

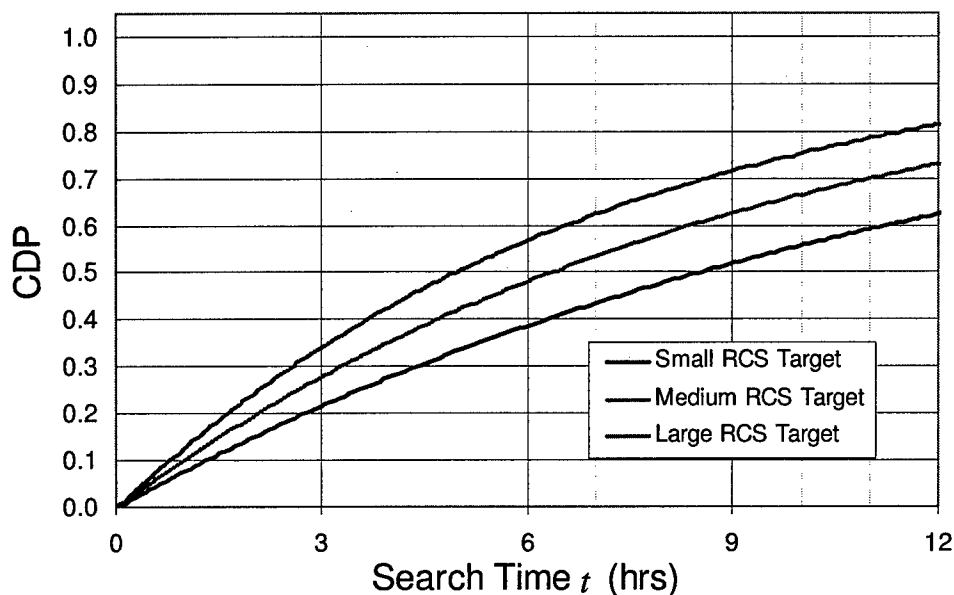


Figure 5. Example: Cumulative Detection Probability as a function of search time

SENSITIVITY ANALYSES AND OPERATIONAL INSIGHTS

Target RCS

The example illustrated in figure 5 shows a significant spread for the resulting CDP due to differences in exposed periscope height of merely .1 meters. Generally, even if excellent information is available concerning enemy submarine periscope mast or periscope RCS, any variability at all will impact the accuracy of the CDP numbers generated with the model.

Effects of Aircraft Altitude

The analytical model does support important operational insight concerning the effects of aircraft altitude and the search for small RCS targets. In the derivation of the model, it is seen that low aircraft altitude improves ARSDS detection rate two ways:

- Low altitude increases the maximum detection range against small RCS targets, and
- Low altitude shortens the distance to the radar horizon, and thus reduces the chance that a submarine can take advantage of a counter-detection

Unfortunately, low altitude also does one other thing, well known to aviators. Low altitude decreases aircraft fuel efficiency thus reducing flight endurance, which shortens the search time on-station, and consequently results in a lower CDP for each aircraft sortie. Therefore, there is a critical operational tradeoff to be made. Shorter flight endurance at higher detection rate, or longer flight endurance as lower detection rate. Values will depend on specific search aircraft and radar characteristics, and the model could be used to find an optimal altitude.

Sea-State Degradation of RCS

Another important operational insight derived from the model concerns the effect of sea-state. As described, for fixed periscope exposure height, increasing sea-state has the effect of decreasing target RCS. Decreased RCS shortens maximum detection range, causing two penalties:

- First, the reduced RCS shortens maximum range, which shortens sweep width, which by itself diminishes the detection rate.
 - Sweep width is approximately proportional to R_{\max}
- Secondly, the shortened radius of the maximum detection area increases the chance that the submarine can avoid detection entirely due to counter-detection evasion, which causes detection rate to diminish further.
 - Probability sub avoids due to counter-detection is proportional to R_{\max}^2

Combined, detection rate is approximately proportional to R_{\max}^3 . For example, if diminished RCS decreases maximum detection range by 50%, i.e. to 1/2 of the previous maximum detection range, then the detection rate is reduced to $(1/2)^3$ or 1/8th of the previous detection rate. The operational implication of this is that as sea-state increases, the aircraft search plan may need to compensate for the reduced RCS with **much** smaller search areas and **lower** search altitudes.

SUMMARY

Countries around the world possess diesel submarines that could threaten sea-lanes and deny maritime access for joint and coalition warfare including operational logistics support of the warfighters. Airborne radar search for diesel submarines is a critically important joint warfare problem. The detection rate model presented for ARSDS can be used to quickly evaluate alternative tactics under varying conditions in ways that support understanding critical operational trade-offs and contribute to search planning. The ARSDS model is easily implemented in a spreadsheet or with other computational software. It accepts real-world data for target characteristics and radar performance, or, in the absence of actual data, can be used with placeholder approximations.

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AIRBORNE RADAR SEARCH FOR DIESEL SUBMARINES

DESCRIPTOR LIST

search and detection
detection rate
radar search
submarine periscope
airborne radar
radar counter-detection
intermittent target